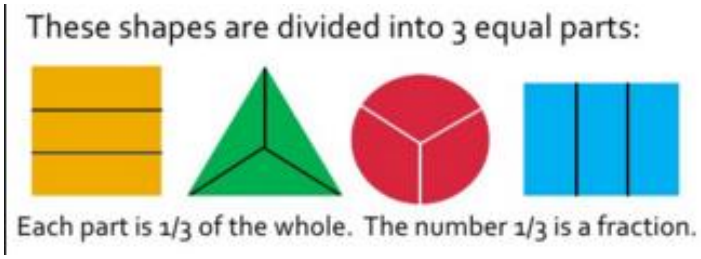
Fractions

Fractions are confusing because they have lots of different meanings. They can be both numbers (e.g. Show me ½ on the number line) and operators (e.g. Find ½ of 12).

Some different meanings of fractions:

An equal part of a whole unit calculation



Result of a division

3 apples shared between 4 people = ¾

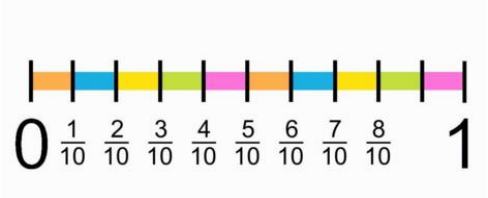
Compare part of a set and the whole set

One out of the three balls is yellow [Proportion]

Compare the sizes of two sets

Pastry contains flour and butter in the ratio 2:3

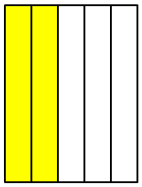
A point between two whole numbers



Equivalent Fractions

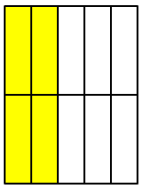
We say two fractions are **equivalent** when they have same value, different name.

Here we have a whole split into 5 parts (fifths) with two parts coloured in:



$\frac{2}{5}$

Here we have a whole split into 10 parts (tenths) with 4 parts coloured in:



$\frac{2}{5} = \frac{4}{10}$

Can you still see $\frac{2}{5}$?

So: $\frac{2}{5}$ is equivalent $\frac{4}{10}$ to

The same amount of the whole is coloured in; we just give it another name.

Adding and Subtracting Fractions with the Same Denominator

Can you work out the following: 3+5=? 3 tens + 5 tens = ? 3 cars + 5 cars = ?
But what about this: 3 bananas + 5 apples = ?

This is only ever 3 bananas and 5 apples; I cannot combine them in the same Way that I could with the others. Why?

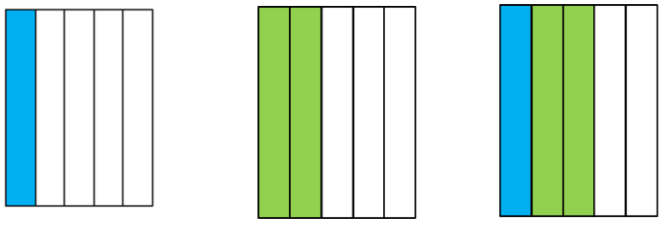
You can only add (or subtract) with the same nouns.

With fractions, our 'noun' is the denominator (bottom number of the fraction).

Example

Here we have wholes split into 5 equal parts (fifths) so we are adding fifths:

One $\frac{1}{5}$ and two $\frac{1}{5}$ equals three $\frac{1}{5}$



$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

"one fifth add two fifths is equal to three fifths"

Subtracting fractions with the same denominators works exactly the same.

Adding and Subtracting Fractions with Different Denominators

You can only add (or subtract) with the same nouns.

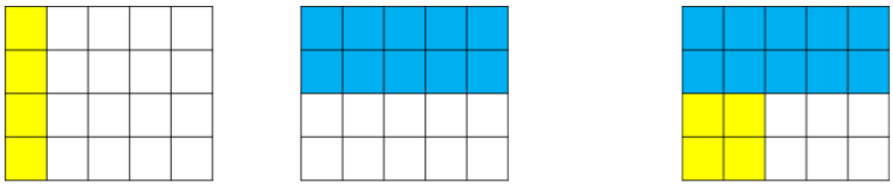
So how would I work this out?

$\frac{1}{5} + \frac{2}{4}$

In order to add these fractions, they need to have the same denominator.
I need to find equivalent fractions.

I need to have a whole that can be split into both fifths and quarters - five equal parts and four equal parts. Well, 5 x 4 = 20 so let's use twentieths (twenty equal parts).

$\frac{1}{5} + \frac{2}{4} = \frac{14}{20}$



$\frac{4}{20} + \frac{10}{20} = \frac{14}{20}$

Similarly, before you can subtract fractions with different denominators you need to find a common denominator.

Multiplying a fraction by a whole number

This is best explained with an example:

$$3 \times \frac{2}{9} = ?$$

Think of this as "3 lots of two-ninths".
In other words:

$$3 \times \frac{2}{9} = \frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{6}{9}$$

Once we understand the concept, there is a quick trick we can use:

Multiply the numerator by the whole number.
The denominator stays the same.

$$3 \times \frac{2}{9} = \frac{6}{9}$$

Now,

$$24 \times \frac{1}{4} = \frac{24}{4} = 6$$ (if we simplify our answer)

Consider,

$\frac{1}{4}$ of 24

To work this out, we can imagine sharing 24 sweets between 4 people (24÷4) = 6

So, finding a fraction of a number is the same as multiplying the number and the fraction…

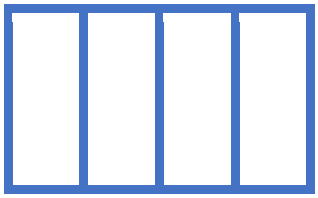
$$24 \times \frac{1}{4} = \frac{1}{4} \times 24 = \frac{1}{4} \text{ of } 24$$

Multiplying fractions (Easier)

Example: $\frac{1}{4} \times \frac{1}{3}$

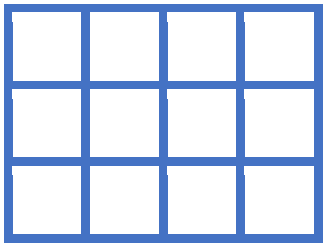
Step 1: When dealing with fractions we always need to first consider our "whole"

Step 2: My first fraction to multiply is $\frac{1}{4}$ so I need to split my whole into 4 equal parts



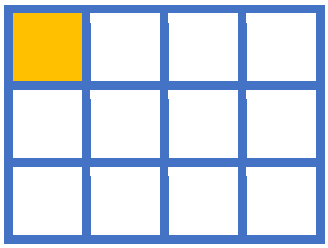
Step 3: Now split each of those quarters into 3 equal parts (since my other fraction is $\frac{1}{3}$)

How many equal parts do I have now? 12



Step 4: I want one of those thirds (one of the three equal parts) and one of those quarters (one of the four equal parts)

So how many twelfths have I got? 1



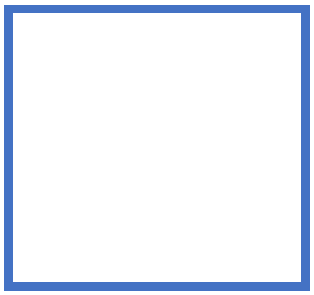
So: $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Quick rule: Multiply the numerators together and multiply the denominators together

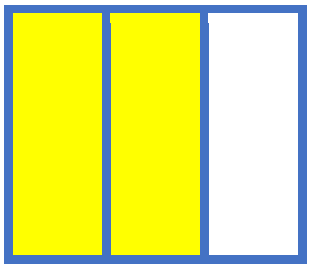
Multiplying fractions (Harder)

Example: $\frac{2}{3} \times \frac{3}{5}$

Step 1: When dealing with fractions we always need to first consider our "whole"

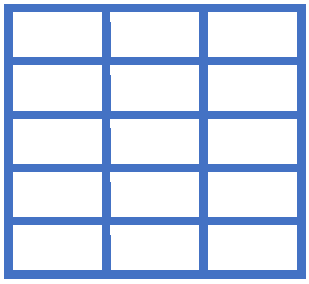


Step 2: My first fraction to multiply is $\frac{2}{3}$ so I need to split my whole into 3 equal parts and I want 2 of them



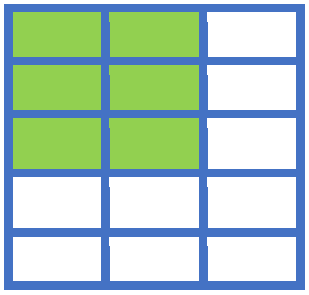
Step 3: Now split each of those thirds into 5 equal parts (since my other fraction $\frac{3}{5}$ has a denominator of 5)

How many equal parts do I have now? 15



Step 4: Just looking at the $\frac{2}{3}$ of my shape (that I coloured yellow above), I want three of those 5 equal pieces (as my fraction is $\frac{3}{5}$)

So how many fifteenths have I got? 6



So: $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

Quick rule: Multiply the numerators together and multiply the denominators together

Dividing a fraction by a whole number

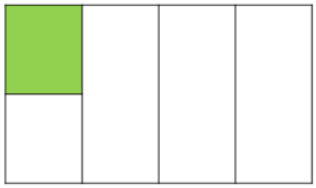
Example: $\frac{1}{4} \div 2$

Step 1: First we must specify what our 'whole' is

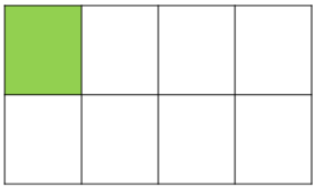
Step 2: I need to split my whole into four equal parts

Step 3: My fraction is $\frac{1}{4}$ so I want one of my four equal parts

Step 4: Let's divide by quarter by 2 (split it into two bits)



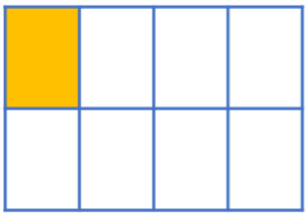
Step 5: In order to know what to call my new part, all the parts must be the same size. When we divide all the quarters by 2, we have 8 equal parts altogether.
That means my green section is $\frac{1}{8}$



So $\frac{1}{4} \div 2 = \frac{1}{8}$

Let's make a connection…

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$



$\times \frac{1}{2}$ is the same as $\div 2$